1. You are given two strings $X[1..m]$ and $Y[1..n]$. Consider the following recursive program, which computes the length of the longest common sub sequence of $X$ and $Y$.

```
PROGRAM longest-common-sub-sequence
for $i := 1$ to $m$ do
    for $j := 1$ to $n$ do
        $l[i, j] := \infty$
    od
od
write('Length =', lcs(m, n))
```

```
FUNCTION lcs(i, j): REAL
if $i \times j = 0$ then
    return 0
else
    if $l[i, j] = \infty$ then
        if $X[i] = Y[j]$ then
            $l[i, j] := lcs(i - 1, j - 1) + 1$
        else
            $l[i, j] := max(lcs(i - 1, j), lcs(i, j - 1))$
        fi
    fi
    return $l[i, j]$
fi
```

Describe how this program works and show its correctness. Further, analyze its runtime and memory requirements and compare these requirements to the ones of the efficient algorithm presented in the lecture. Explain in detail why your analysis is correct.

**Remark:** Note that $l$ is a matrix of size $[1..m, 1..n]$.