Advantages of mutation in passive testing: An empirical study.
Mutation 2009

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Outline

1. Motivation
2. Theoretical Aspects
3. PASTE
4. Case study
Formal Testing

Formal testing techniques are used to check the correct behaviour of systems.

What do they do?
Functional Behavior

How do they do?
Non-Functional Properties

In real-time systems the temporal behavior is considered critical.
Formal Testing

Active Testing
Testers are allowed to interact directly with the system.

Passive Testing
Testers analyze traces extracted from the system.

Our approach
- Framework to perform passive testing in systems with temporal restrictions
- Applied on a formal model: Finite State Machines.
Timed Finite State Machines

\[ F_2(x) = \begin{cases} 
0 & x < 0 \\
\frac{x}{2} & 0 \leq x < 2 \\
1 & 2 \leq x 
\end{cases} \]

Uniform distribution in \([0, 2)\)
Common conditions

Both specification and implementations:

- Expressed by $\text{TFSM}$. 
- Input-enabled. 
- Regular stochastic information. 
- Observable.
Common conditions

Both specification and implementations:

- Expressed by \( \text{TFSM} \).
- Input-enabled.
- Regular stochastic information.
- Observable.

Regular stochastic information

\[
\begin{array}{c}
\overset{x_3/y_2, F_3}{\bullet} \\
\overset{x_1/y_1, F_1}{\bullet} & \overset{x_1/y_2, F_2}{\bullet} & \overset{x_1/y_2, F_2}{\bullet}
\end{array}
\]
Common conditions

Both specification and implementations:

- Expressed by TFSM.
- Input-enabled.
- Regular stochastic information.
- Observable.

Observable: $x_1/y_1, F_1$
Non-Observable: $x_1/y_2, F_2$

Observable: $x_1/y_1, F_1$
Non-Observable: $x_1/y_1, F_1$
Timed Invariants

Allow us to express temporal properties that must be fulfilled by the implementation.

- Extracted from the specification.
- Given by the tester from the original requirements.

Correctness

1. Time invariants are correct with respect to the specification.
2. Invariants are satisfied by the traces produced by the implementation.
Timed Invariants

\[
\begin{align*}
\phi_1 &= i_1 \mapsto \{ \langle o_2, F_3 \rangle \} \\
\phi_2 &= i_2 \mapsto \{ \langle o_1, F_3 \rangle, \langle o_3, F_2 \rangle \} \\
\phi_3 &= i_1 / ? / F_3, i_1 \mapsto \{ \langle o_2, F_3 \rangle \} \\
\phi_4 &= i_1 / o_2 / F_3, i_2 / o_3 / F_2, *, i_0 \mapsto \{ \langle o_1, F_1 \rangle \}
\end{align*}
\]
PASsive TEsting (PASTE)

- Automates our passive testing approach.
- Implements the algorithms defined in our framework.
PASsive TEsting (PASTE)

- Some invariants have more power than others for detecting faults.
  - Length of the invariant.
  - Length of the trace.
  - Kind of fault.

- Selection of the best invariants.
- Functionality based on mutation techniques.
Mutants Module

Help the tester to select the most efficient invariants to capture the different errors.

1. Generation of mutants from the specification.
2. Extraction of traces from the produced mutants.
3. Application of invariants to the extracted traces.
4. Analysis of the results for determining the level of effectiveness of the invariants.
Changing an output

The mutant must be observable.

The mutant must present regular stochastic information.
Changing an output

The mutant must be observable.
The mutant must present regular stochastic information.
Changing the target state in a transition

$x_1/y_1, F_3$
$x_1/y_2, F_1$
$x_1/y_3, F_2$
$x_1/y_2, F_1$

$X$
Changing the target state in a transition

\[ \begin{align*}
x_1/y_1, F_3 & \quad x_1/y_2, F_1 \\
x_1/y_3, F_2 & \\
& \quad x_1/y_2, F_1
\end{align*} \]
Altering the distribution function

The mutant must present regular stochastic information.
Alter the distribution function

\[ \frac{x_1}{y_1}, F_1 \]

\[ \frac{x_1}{y_1}, F_2 \]

\[ \frac{x_1}{y_1}, F_1 \]

\[ \frac{x_1}{y_2}, F_2 \]

\[ \frac{x_1}{y_2}, F_2 \]

\[ \frac{x_1}{y_1}, F_2 \]

The mutant must present regular stochastic information.
The specification

\[ F_1(t) = \begin{cases} 0 & \text{if } t < 0 \\ \frac{t}{2} & \text{if } 0 \leq t < 2 \\ 1 & \text{if } 2 \leq t \end{cases} \]

\[ F_2(t) = \begin{cases} 0 & \text{if } t < 4 \\ 1 & \text{if } 4 \leq t \end{cases} \]

\[ F_3(t) = \begin{cases} 1 - e^{-\frac{t}{3}} & \text{if } 0 \leq t \\ 0 & \text{if } t > 0 \end{cases} \]
Set of Invariants

\[ \phi_1 = \begin{align*}
& i_2 \mapsto \{(o_9, F_1), (o_8, F_2), (o_7, F_1), (o_6, F_3), (o_5, F_3), (o_4, F_2), (o_2, F_1)\}\end{align*} \]

\[ \phi_2 = \begin{align*}
& i_2 \mapsto \{(o_9, F_1), (o_8, F_2), (o_7, F_1), (o_6, F_3), (o_5, F_3), (o_4, F_2), (o_2, F_1), (o_1, F_1)\}\end{align*} \]

\[ \phi_3 = \begin{align*}
& i_2 \mapsto \{(o_9, F_1), (o_8, F_2), (o_7, F_1), (o_6, F_3), (o_5, F_3), (o_4, F_2), (o_3, F_1), (o_2, F_1)\}\end{align*} \]

\[ \phi_4 = \begin{align*}
& i_2 \mapsto \{(o_9, F_1), (o_8, F_2), (o_7, F_1), (o_6, F_3), (o_5, F_3), (o_4, F_2), (o_3, F_1), (o_2, F_1), (o_1, F_1)\}\end{align*} \]

\[ \phi_5 = \begin{align*}
& i_2 \mapsto \{(o_{10}, F_3), (o_9, F_1), (o_8, F_2), (o_7, F_1), (o_6, F_3), (o_5, F_3), (o_4, F_2), (o_2, F_1)\}\end{align*} \]

\[ \phi_6 = \begin{align*}
& i_2 \mapsto \{(o_{10}, F_3), (o_9, F_1), (o_8, F_2), (o_7, F_1), (o_6, F_3), (o_5, F_3), (o_4, F_2), (o_2, F_1), (o_1, F_1)\}\end{align*} \]

\[ \phi_7 = \begin{align*}
& i_2 \mapsto \{(o_{10}, F_3), (o_9, F_1), (o_8, F_2), (o_7, F_1), (o_6, F_3), (o_5, F_3), (o_4, F_2), (o_3, F_1), (o_2, F_1)\}\end{align*} \]

\[ \phi_8 = \begin{align*}
& i_2 \mapsto \{(o_{10}, F_3), (o_9, F_1), (o_8, F_2), (o_7, F_1), (o_6, F_3), (o_5, F_3), (o_4, F_2), (o_3, F_1), (o_2, F_1), (o_1, F_1)\}\end{align*} \]

\[ \phi_9 = \begin{align*}
& i_1 \mapsto \{(o_{10}, F_1), (o_{7}, F_3), (o_6, F_3), (o_5, F_3), (o_4, F_2), (o_3, F_1), (o_2, F_1), (o_1, F_1)\}\end{align*} \]

\[ \phi_{10} = \begin{align*}
& i_1 \mapsto \{(o_{10}, F_1), (o_8, F_2), (o_7, F_3), (o_6, F_2), (o_5, F_1), (o_4, F_3), (o_3, F_1), (o_2, F_1), (o_1, F_1)\}\end{align*} \]

\[ \phi_{11} = \begin{align*}
& i_2, o_8/F_2, i_2 \mapsto \{(o_{10}, F_1), (o_8, F_2), (o_7, F_1), (o_6, F_3), (o_5, F_3), (o_4, F_2), (o_2, F_1), (o_1, F_1)\}\end{align*} \]

\[ \phi_{12} = \begin{align*}
& i_2, o_8/F_2, i_1 \mapsto \{(o_{10}, F_1), (o_8, F_2), (o_7, F_1), (o_6, F_2), (o_5, F_1), (o_4, F_3), (o_3, F_1)\}\end{align*} \]

\[ \phi_{13} = \begin{align*}
& i_2, o_8/F_2, i_1 \mapsto \{(o_{10}, F_1), (o_8, F_2), (o_7, F_1), (o_6, F_2), (o_5, F_1), (o_4, F_3), (o_3, F_1), (o_1, F_1)\}\end{align*} \]

\[ \phi_{14} = \begin{align*}
& i_2, o_8/F_2, i_2 \mapsto \{(o_{10}, F_3), (o_8, F_2), (o_7, F_1), (o_6, F_3), (o_5, F_3), (o_4, F_2), (o_3, F_1), (o_2, F_1)\}\end{align*} \]

\[ \phi_{15} = \begin{align*}
& i_1, o_3/F_1, i_2 \mapsto \{(o_{9}, F_1), (o_7, F_1), (o_5, F_3), (o_4, F_2), (o_3, F_1), (o_2, F_1), (o_1, F_2)\}\end{align*} \]

\[ \phi_{16} = \begin{align*}
& i_1, o_3/F_1, i_1 \mapsto \{(o_{9}, F_3), (o_7, F_3), (o_5, F_1), (o_4, F_3), (o_3, F_1), (o_2, F_3), (o_1, F_1)\}\end{align*} \]

\[ \phi_{17} = \begin{align*}
& i_2, o_8/F_2, i_1, o_3/F_1, i_2 \mapsto \{(o_{9}, F_1), (o_8, F_2), (o_7, F_1), (o_6, F_3), (o_5, F_3), (o_4, F_2), (o_2, F_1)\}\end{align*} \]

\[ \phi_{18} = \begin{align*}
& i_2, o_8/F_2, i_1, o_3/F_1, i_2 \mapsto \{(o_{9}, F_1), (o_8, F_2), (o_7, F_1), (o_6, F_3), (o_5, F_3), (o_4, F_2), (o_2, F_1), (o_1, F_1)\}\end{align*} \]

\[ \phi_{19} = \begin{align*}
& i_2, o_8/F_2, i_1, o_3/F_1, i_1 \mapsto \{(o_{10}, F_1), (o_7, F_3), (o_6, F_2), (o_5, F_1), (o_4, F_3), (o_3, F_1), (o_2, F_3), (o_1, F_1)\}\end{align*} \]

\[ \phi_{20} = \begin{align*}
& i_2, o_8/F_2, *, i_1 \mapsto \{(o_{10}, F_1), (o_9, F_1), (o_7, F_3), (o_6, F_2), (o_5, F_1), (o_4, F_3), (o_3, F_1), (o_2, F_3), (o_1, F_1)\}\end{align*} \]

\[ \phi_{21} = \begin{align*}
& i_1, o_3/F_1, i_1, o_3/F_1, i_1 \mapsto \{(o_{10}, F_1), (o_8, F_2), (o_7, F_3), (o_6, F_3), (o_5, F_1), (o_4, F_3), (o_3, F_1), (o_2, F_3), (o_1, F_1)\}\end{align*} \]
Motivation

Theoretical Aspects

Non equivalent mutants

<table>
<thead>
<tr>
<th></th>
<th>CO</th>
<th>CGS</th>
<th>CT</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>201</td>
<td>230</td>
<td>207</td>
<td>638</td>
</tr>
</tbody>
</table>

Case study

Mutants generated
- All mutants were generated for CO and CGS operators.
- Different levels of deviation for CT operator.

Traces extracted
- From each mutant 10 traces are extracted.
- Different length: \( k \cdot |S| \) where \( k \in \{1..10\} \)

Analysis of results
- Length of traces
- Length of the invariants.
- Kind of mutant from which the trace was extracted.
Results I: Output mutants

<table>
<thead>
<tr>
<th>INV Length 1</th>
<th>INV Length 2</th>
<th>INV Length 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>3.68%</td>
<td>11.69%</td>
</tr>
<tr>
<td>9</td>
<td>3.53%</td>
<td>11.09%</td>
</tr>
<tr>
<td>8</td>
<td>3.03%</td>
<td>10.70%</td>
</tr>
<tr>
<td>7</td>
<td>2.54%</td>
<td>10.45%</td>
</tr>
<tr>
<td>6</td>
<td>2.44%</td>
<td>10.25%</td>
</tr>
<tr>
<td>5</td>
<td>1.54%</td>
<td>8.31%</td>
</tr>
<tr>
<td>4</td>
<td>1.44%</td>
<td>7.71%</td>
</tr>
<tr>
<td>3</td>
<td>1.04%</td>
<td>6.57%</td>
</tr>
<tr>
<td>2</td>
<td>0.57%</td>
<td>5.72%</td>
</tr>
<tr>
<td>1</td>
<td>3.38%</td>
<td>4.45%</td>
</tr>
</tbody>
</table>
Results II: Transfer mutants

-INV Length 2
-INV Length 3
Results III: Time mutants
Measuring effectiveness

\[
\frac{w_{CO} \cdot M_{CO}(\phi_i) + w_{CT} \cdot M_{CT}(\phi_i) + w_{CGS} \cdot M_{CGS}(\phi_i)}{NumTracesMut \cdot M_{TOT}}
\]

where \( \sum_{i \in \{CO, CT, CGS\}} w_i = 1 \)

- \( M_{CO}(\phi_i), M_{CGS}(\phi_i) \) and \( M_{CT}(\phi_i) \): number of errors detected by the invariant \( \phi_i \).
- \( M_{TOT} \): number of mutants generated.
- \( NumTracesMut \): number of traces extracted from each mutant.
- \( w_{CO}, w_{CT} \) and \( w_{CGS} \) are selected by the tester.
Measuring effectiveness

### Set of Invariants

<table>
<thead>
<tr>
<th>$w_{co}$</th>
<th>$w_{ct}$</th>
<th>$w_{cgs}$</th>
<th>$\phi_1 &gt; \phi_3 &gt; \phi_2 &gt; \phi_5 &gt; \phi_9 &gt; \phi_{15} &gt; \phi_{12} &gt; \phi_{16} &gt; \phi_4 \simeq \phi_7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{3}$</td>
<td>$\frac{1}{3}$</td>
<td>$\frac{1}{3}$</td>
<td>(\phi_1 &gt; \phi_3 &gt; \phi_2 &gt; \phi_5 &gt; \phi_9 &gt; \phi_{15} &gt; \phi_{12} &gt; \phi_{16} &gt; \phi_4 \simeq \phi_7)</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>(\phi_1 &gt; \phi_3 &gt; \phi_2 &gt; \phi_5 &gt; \phi_9 &gt; \phi_4 &gt; \phi_7 &gt; \phi_6 &gt; \phi_{10} &gt; \phi_{16})</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>(\phi_{15} &gt; \phi_{12} &gt; \phi_{18} &gt; \phi_{17} &gt; \phi_{16} &gt; \phi_{14} \simeq \phi_{19} &gt; \phi_{21} &gt; \phi_{11} &gt; \phi_{13})</td>
</tr>
</tbody>
</table>
Conclusions

- Application of mutation techniques to \textsc{PASTE} tool in order to check the effectiveness of the invariants for detecting errors.
- Mutation operators for obtaining mutants from the specification.
- Different traces are extracted from the mutants that simulate real faults.
- The traces are used to test if the invariants proposed by the tester can find the errors and a measure of its effectiveness is estimated.
Thanks for your attention!