Interactive Course Material for Signal Processing based on Isabelle/\texttt{ISAC}

Baccalaureate Thesis

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September 2, 2011
1. Introduction
2. Fourier transformation
   Fourier transform
3. LTI systems
   Convolution (Faltung)
4. Z-Transform
   (Inverse) Z-Transform
5. Conclusions
Issues to be Accomplished

- What knowledge is already mechanised in *Isabelle*?
- How can missing theorems and definitions be mechanised?
- What is the effort for such mechanisation?
- How do calculations look like, by using mechanised knowledge?
- What problems and subproblems have to be solved?
- Which problems are already implemented in *ISAC*?
- How are the new problems specified (*ISAC*)?
- Which variants of programs in *ISAC* solve the problems?
- What is the contents of the interactive course material (Figures, etc.)?
1 Introduction

2 Fourier transformation
   Fourier transform

3 LTI systems
   Convolution (Faltung)

4 Z-Transform
   (Inverse) Z-Transform

5 Conclusions
Fourier Transformation: Introduction

Possibilities:
- Transform operation by using property-tables
- Transform operation by using integral

Also Important:
- Visualisation?!
Fourier Transformation: Specification

Determine the fourier transform for the given rectangular impulse:

\[ x(t) = \begin{cases} 
1 & -1 \leq t \leq 1 \\
0 & \text{else} 
\end{cases} \]

given : piecewise_function

\[ \text{fun}(x(t :: \text{real}), x = 1 ((t \geq -1) \& (t \leq 1)), x = 0) \]

precond : TODO

find : \( X(j \cdot \omega) \)

postcond : TODO
## Fourier Transform: Development effort

<table>
<thead>
<tr>
<th>requirements</th>
<th>comments</th>
<th>effort</th>
</tr>
</thead>
<tbody>
<tr>
<td>solving Integrals</td>
<td>simple via properties table <em>real</em></td>
<td>20 MT</td>
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<td>transformation table</td>
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<td>visualisation</td>
<td>backend</td>
<td>10</td>
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<tr>
<td>example collection</td>
<td>with explanations</td>
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<td>70-80</td>
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</tbody>
</table>

Effort — in 45min units

MT — thesis “Integrals” (mathematics)
Fourier Transformation: Summary

- Standard integrals can be solved with tables
- No real integration (yet available)
- Math tricks difficult to implement
Outline

1. Introduction
2. Fourier transformation
   Fourier transform
3. LTI systems
   Convolution (Faltung)
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   (Inverse) Z-Transform
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Convolution: Introduction

- Calculation include sums
- Demonstrative examples
- Visualisation is important
Convolution: Specification

Consider the two discrete-time, linear and time-invariant (LTI) systems with the following impulse response:

\[ h_1[n] = \left(\frac{3}{5}\right)^n \cdot u[n] \]
\[ h_1[n] = \left(-\frac{2}{3}\right)^n \cdot u[n] \]

The two systems are cascaded seriell. Derive the impulse response of the overall system \( h_c[n] \).

given : Signals \( h_1[n], h_2[n] \)
\((h_1[n]=(3/5)^n*u[n]), h_2[n]=(-2/3)^n*u[n])\)

precond : TODO
find : \( h_1[n] \cdot h_2[n] \)
postcond : TODO
**Convolution: Development effort**

<table>
<thead>
<tr>
<th>requirements</th>
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<th>effort</th>
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<tbody>
<tr>
<td>simplify rationals</td>
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<td>example collection</td>
<td>with explanations</td>
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</tbody>
</table>

effort — in 45min units
Convolution: Summary

- Standard example
- Straight forward
- Challenge are sum limits
(Inverse) \( \mathcal{Z} \)-Transformation: Introduction

- Pure Transformation is simple to realise with Z-Transform Properties (Table)
- Partial Fraction are just math simplifications
(Inverse) Z-Transformation: Specification

Determine the inverse z transform of the following expression. Hint: apply the partial fraction expansion.

\[ X(z) = \frac{3}{z - \frac{1}{4} - \frac{1}{8} z^{-1}} \], \quad x[n] \text{ is absolute summable}

given : Expression of z
\( (X(z::\text{complex}), 3/(z-1/4-1/8 z^{-1})) \)
precond : TODO
find : Expression of n
\( h[n] \)
postcond : TODO
## (Inverse) Z-Transformation: Development effort

<table>
<thead>
<tr>
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<th>effort</th>
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</tr>
<tr>
<td>example collection</td>
<td>with explanations</td>
<td>20</td>
</tr>
</tbody>
</table>

effort — in 45min units

MT — thesis “factorization” (mathematics)
(Inverse) Z-Transformation: Summary

- No *higher* math operations
- Different subproblems of math (equation systems, etc.)
- Both directions have the same effort
Demonstration
Conclusions

Design Challenges:

- Pre and Post conditions
- Exact mathematic behind functions
- Accurate mathematic notation

Goals:

- Spot the power of ISAC
- Implementation of generell but simple math problems
- Setting up a good first guideline (documentation) for further problem implementations

Efforts are only approximations, due we have no real experience data!
Comming up

Juli 2011
project startup
Juli 2011
information collection, 1st presentation
August 2011
extern traineeship
September 2011
main work
after Oktober
finishing, documentation