Mechanised Explanation in "Systems that Explain Themselves"
Magic caused by Confusion of Languages?

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CME-EI: Computer Mathematics in Education — Enlightenment or Incantation?
at CICM, Hagenberg, Austria
Aug. 17, 2018
1. Magic caused by Confusion of Languages?

2. Mechanical Explanation and Language Layers
   - Term Language
   - Proof Language
   - Specification Language
   - “Next step guidance”
   - Programming Language

3. Conclusions?
## Confusion of Languages?

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  - with derivation in (formal) logic
  - What does language communicate?
    - emotions and explanations
    - formal content
  - How is self-referentiality handled?
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Confuse Languages?

Languages?

Term Language

Proof Language

Specification

Step Guidance

Prog. Language

Conclusions
1. Magic caused by Confusion of Languages?

2. Mechanical Explanation and Language Layers
   - Term Language
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   - “Next step guidance”
   - Programming Language

3. Conclusions?
Term Language

The **demo** has shown . . .

- principal benefits
  - uniformity over all domains of mathematics
  - type system efficiently excludes ambiguities
  - explicit description of operations and respective rules

- added value of implementation
  - a formula’s elements are connected with definitions
  - types are transparent by mouse pointer
  - feedback to input of formulas
  - structure of formulas, i.e. sub-terms are transparent
  - representation adaptable to engineers’ needs
The demo has shown . . .

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3 Conclusions?
Proof Language . . .

. . . adapts to conventions of engineering mathematics:

Figure: Conventional worksheet on ISAC’s front-end
Proof Language

- Principal benefits
  - calculations in a conventional format
  - all steps of calculation in a consistent framework
  - each step is justified by theorems
  - specific steps equivalent to Computer Algebra
  - Computer Algebra decomposed into elementary steps

- Added value of implementation
  - change from survey to detail in the calculation tree (collapsing and expanding)
  - justification for any step can be inspected on demand
  - steps can be redone while trying alternative ways
  - alternatives can be tried in parallel windows
Proof Language

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3. Conclusions?
Formal specification of the previous Solution:

01 Problem (Biegelinie, [Biegelinien])
02 Specification:
03 Model:
04 Given : Traegerlaenge L, Streckenlast q₀
05 Where : q₀ ist_integrierbar_auf [0, L]
06 Find : Biegelinie y
07 Relate : Randbedingungen [Q 0 = q₀ · L, M₀ L = 0, y 0 = 0, \( \frac{d}{dx} y 0 = 0 \)]
08 References:
09 Theory : Biegelinie
10 x Problem : ["Biegelinien"]
11 o Method : ["IntegrierenUndKonstanteBestimmen2"]
12 Solution:

Hidden data for “next step guidance”:

[ ( [ Traegerlaenge L, Streckenlast q₀, Biegelinie y, Randbedingungen [ Q 0 = q₀ · L, M₀ L = 0, y 0 = 0, \( \frac{d}{dx} y 0 = 0 \)], FunktionsVariable x ] ) ]
Specification Language

- Principal benefits
  - formal specification prepares mechanical solution
  - pre-condition determines solvability
  - post-condition makes essence of a problem explicit
  - problems decomposed into sub-problems (with specifications)

- Added value of implementation
  - specifications can be easily searched and tried
  - trees of specifications allow automated refinement
  - successfully specified problems solved by key stroke
  - sub-problems can be interactively arranged
  - specifications as black boxes raises abstraction in problem solving, see slide movie
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Start Example

*Problem [area-of-circle]*

*Problem [area-of-circle]*

*Example 1*

From a horizontally lying pipe with a diameter of 8 cm there are 5 liters of water flowing out per second. At what height is this pipe, if the horizontal distance between outlet and incidence on the floor is 80 cm?

**Note:** First determine the exit velocity (by use of the volume of water per second and of the cross-section area.)

\[ d \]
Start Example

From a horizontally lying pipe with a diameter of 8 cm there are 5 liters of water flowing out per second. At what height is this pipe, if the horizontal distance between outlet and incidence on the floor is 80 cm?
Note: First determine the exit velocity (by use of the volume of water per second and of the cross-section area.)
Problem modelled ok
Start considering sub-problems
Select sub-problems

Problem [rational, equation]

Find: $h \text{ m}$
Select sub-problems

Model:
- Given: Diameter $d = 8$ cm, FlowRate $\phi = 54$ l/s,
  HorizontalDistance $s = 80$ cm
- Where: $d > 0 \land \phi > 0 \land s > 0$
- Find: HeightOfPipe $h$

Problem [rational, equation]

Problem [velocity-space-time, find-time]

Find: $h$ m
Select sub-problems

Problem [rational, equation]

\[ v = \frac{s}{t} \]

Problem [velocity-space-time, find-time]

Problem [flow-rate, find-velocity]

Find: \( h \) m

\[ d \text{ cm, } \phi \text{ l/s, } s \text{ cm} \]
Select sub-problems

- d cm, φ l/s, s cm
- Problem [rational, equation]
- Problem [velocity-space-time, find-time] \( v = \frac{s}{t} \)
- Problem [flow-rate, find-velocity] \( v = \frac{\phi}{A_{\text{circle}}} \)
- Problem [free-fall] \( h = \frac{g}{2} \cdot t^2 \)

Find: \( h \) m
Delete irrelevant sub-problems

Problem [rational, equation]

Problem [flow-rate, find-velocity]

Problem [velocity-space-time, find-time]

Problem [free-fall]

Find: \( h \) m

Problem [area-of-circle]
Select relevant sub-problems

- **Problem [area-of-circle]**
  \[ A_{\text{circle}} = \left( \frac{d}{2} \right)^2 \cdot \pi \]

- **Problem [velocity-space-time, find-time]**
  \[ \nu = \frac{s}{t} \]

- **Problem [flow-rate, find-velocity]**
  \[ \nu = \frac{\phi}{A_{\text{circle}}} \]

- **Problem [free-fall]**
  \[ h = \frac{g}{2} \cdot t^2 \]
What is given / has to be found?

- Problem [area-of-circle]
  \[ A_{\text{circle}} = \left( \frac{d}{2} \right)^2 \cdot \pi \]

- Problem [velocity-space-time, find-time]
  \[ \nu = \frac{s}{t} \]

- Problem [flow-rate, find-velocity]
  \[ \nu = \frac{\varphi}{A_{\text{circle}}} \]

- Problem [free-fall]
  \[ h = \frac{g}{2} \cdot t^2 \]

Find: \( h \) m

Given: Diameter \( d = 8 \text{ cm} \), FlowRate \( \varphi = 54 \text{ l/s, } \)
HorizontalDistance \( s = 80 \text{ cm} \)
Where: \( d > 0 \land \varphi > 0 \land s > 0 \)
Find: \( \text{HeightOfPipe} h \)
What is given / has to be found?

- Diameter \( d = 8 \, \text{cm} \), flow rate \( \phi = 54 \, \text{l/s} \), horizontal distance \( s = 80 \, \text{cm} \)

Find:
- Height of pipe \( h \)
Connect “Given” and “Find”

Problem [flow-rate, find-velocity]
\[ v = \frac{\phi}{A_{circle}} \]

Problem [velocity-space-time, find-time]
\[ v = \frac{s}{t} \]

Problem [free-fall]
\[ h = \frac{g}{2} \cdot t^2 \]

Find: \( h \) m

Problem [area-of-circle]
\[ A_{circle} = \left( \frac{d}{2} \right)^2 \cdot \pi \]

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Where: \( d > 0 \land \phi > 0 \land s > 0 \)
Find: HeightOfPipe \( h \)
Connect “Given” and “Find”

Problem [flow-rate, find-velocity]

Problem [velocity-space-time, find-time]

Problem [free-fall]

Find: $h$ m

Problem [area-of-circle]
Connect “Given” and “Find”

Model:
Given: Diameter $d = 8 \text{ cm}$, Flow Rate $\phi = 54 \text{ l/s}$,
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Where: $d > 0 \land \phi > 0 \land s > 0$
Find: RightOfPipe $h$

Problem [area-of-circle]

$A_{circle} = \left(\frac{d}{2}\right)^2 \cdot \pi$

Problem [velocity-space-time, find-time]

$v = \frac{s}{t}$

Problem [flow-rate, find-velocity]

$v = \frac{\phi}{A}$

Problem [free-fall]

$h = \frac{g}{2} \cdot t^2$

Find: $h \text{ m}$
Connect “Given” and “Find”

Problem [flow-rate, find-velocity]
\[ \nu = \frac{\phi}{A_{\text{circle}}} \]

Problem [velocity-space-time, find-time]
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\[ h = \frac{g}{2} \cdot t^2 \]

Find: \( h \) m

\[ d \text{ cm}, \ \phi \text{ l/s}, \ s \text{ cm} \]
Connect “Given” and “Find”

Problem [flow-rate, find-velocity]
\[ \nu = \frac{\phi}{A} \]

Problem [velocity-space-time, find-time]
\[ \nu = \frac{s}{t} \]

Problem [area-of-circle]
\[ A_{\text{circle}} = \frac{(d)}{2} \cdot \pi \]

Problem [free-fall]
\[ h = \frac{g}{2} \cdot t^2 \]

Find: \( h \) \text{m}
Dangling connection ???

- Problem [flow-rate, find-velocity]
  \[ \nu = \frac{\phi}{A_{\text{circle}}} \]

- Problem [velocity-space-time, find-time]
  \[ \nu = \frac{s}{t} \]

- Problem [free-fall]
  \[ h = \frac{g}{2} \cdot t^2 \]

Find: \( h \) m

- Problem [area-of-circle]
  \[ A_{\text{circle}} = \left( \frac{d}{2} \right)^2 \cdot \pi \]
Rearrange sub-problems

Problem [area-of-circle]

Problem [velocity-space-time, find-time]

Problem [flow-rate, find-velocity]

Problem [free-fall]

Find: h m

d cm, \( \phi \) l/s, s cm

Problem [flow-rate, find-velocity]

Problem [velocity-space-time, find-time]

Problem [free-fall]

Find: h m

\( A_{circle} = \left(\frac{d}{2}\right)^2 \cdot \pi \)

\( v = \frac{s}{t} \)

\( h = \frac{g}{2} \cdot t^2 \)
Flipped two sub-problems

Find: h m
h  m

Problem [area-of-circle]

Problem [flow-rate, find-velocity]

Problem [velocity-space-time, find-time]

Problem [free-fall]

Find: h m
Connect “Given” and “Find”

Problem [area-of-circle]

\[ A_{circle} = \left( \frac{d}{2} \right)^2 \cdot \pi \]

Find: \( h \) \( m \)

Problem [flow-rate, find-velocity]

\[ v = \frac{\phi}{A_{circle}} \]

\( v \) \( m/s \)

Problem [velocity-space-time, find-time]

\[ v = \frac{s}{t} \]

\( t \) \( s \)

Problem [free-fall]

\[ h = \frac{g}{2} \cdot t^2 \]

Find: \( h \) \( m \)
Connect “Given” and “Find”

Problem [area-of-circle]

\[ A_{\text{circle}} = \left( \frac{d}{2} \right)^2 \cdot \pi \]

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Find: \( h \) m
Connect “Given” and “Find”

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  \[ A_{\text{circle}} = \left(\frac{d}{2}\right)^2 \cdot \pi \]
  - Given: Diameter \( d = 8 \text{ cm} \), FlowRate \( \phi = 8 \text{ l/s} \)
  - Find: \( h \text{ m} \)

- Problem [flow-rate, find-velocity]
  \[ v = \frac{\phi}{A_{\text{circle}}} \]
  - \( v \text{ m/s} \)
  - \( s \text{ cm} \)

- Problem [velocity-space-time, find-time]
  \[ v = \frac{s}{t} \]
  - \( t \text{ s} \)

- Problem [free-fall]
  \[ h = \frac{g}{2} \cdot t^2 \]
  - \( h \text{ m} \)
Connect “Given” and “Find”

Problem [area-of-circle]

\[ A_{\text{circle}} = \left(\frac{d}{2}\right)^2 \pi \]

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Problem [velocity-space-time, find-time]

\[ v = \frac{s}{t} \]

Problem [free-fall]

\[ h = \frac{g}{2} \cdot t^2 \]

Find: \( h \) m

\( d \) cm, \( \phi \) l/s, \( s \) cm

\( v \) m/s

\( h \) m

\( s \) cm

\( \phi \) m3/s
Connect “Given” and “Find”

Mechanised Explanation
Walther Neuper

Confuse Languages?
Lang. Layers
Term Language
Proof Language
Specification
Step Guidance
Prog. Language

Conclusions
All connections finished
Care about unit conversions

Problem [area-of-circle]

\[ A_{\text{circle}} = \left(\frac{d}{2}\right)^2 \cdot \pi \]

Problem [flow-rate, find-velocity]

\[ v = \frac{\phi}{A_{\text{circle}}} \]

Problem [velocity-space-time, find-time]

\[ v = \frac{s}{t} \]

Problem [free-fall]

\[ h = \frac{g}{2} \cdot t^2 \]

Find: \( h \) m

\( d \) cm, \( \phi \) l/s, \( s \) cm

\( \pi \) cm, \( \phi \) m3/s

\( s \) cm, \( s \) m

Find: \( h \) m

Care about unit conversions

Mechanised Explanation
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Confuse Languages?
Lang. Layers
Term Language
Proof Language
Specification
Step Guidance
Prog. Language

Conclusions
Solution with units only

Solution:

Problem [area-of-circle]:

\[ A_{\text{circle}} \text{ cm} \]
\[ A_{\text{circle}} \text{ m} \]
\[ \phi = 5 \text{ cm/s} \]
\[ \phi = 0.005 \text{ m/s} \]

Problem [flow-rate, find-velocity]:

\[ v \text{ m/s} \]
\[ s = 80 \text{ cm} \]
\[ s = 0, 8 \text{ m} \]

Problem [velocity-space-time, find-time]:

\[ t \text{ m/s} \]
\[ t \text{ m} \]

Problem [free-fall]:

\[ h \text{ m} \]
Solution complete
1. Magic caused by Confusion of Languages?

2. Mechanical Explanation and Language Layers
   - Term Language
   - Proof Language
   - Specification Language
   - “Next step guidance”
   - Programming Language

3. Conclusions?
“Next step guidance” . . .

• ... in specifying a problem:

If **data** for each variant **for constructing a specification** (one variant shown above) are given, then the system can guide the student in completing a specification

• ... in step-wise constructing a solution:

If a **program** describes how to solve a problem defined by a formal specification, then this program run by **Lucas-Interpretation**

  • determines a next step (if requested by the student)
  • checks input of the student using the logical context.
“Next step guidance” . . .

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  If a **program** describes how to solve a problem defined by a formal specification, then this program run by **Lucas-Interpretation**
    • determines a next step (if requested by the student)
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Outline

1. Magic caused by Confusion of Languages?

2. Mechanical Explanation and Language Layers
   - Term Language
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3. Conclusions?
Programming Language . . .

... for authors of mathematics knowledge.

**Isabelle** provides a “function package” for programming. Added value of this implementation:

- syntax errors are indicated accurately
- type annotations shift into the initial signature
- less type annotations are required
- syntax highlighting specific for constants etc
- free variables on right-hand-sides are rejected

Students might watch progress within a solution like in a debugger (on request).
Conclusions?

human/natural language $\neq$ formal language of mathematics

respective differences are not clarified in education which . . .

Assumption: . . . misleads to conceiving mathematics as magic, i.e. to inappropriate expectations about math.

Safe assumption: these differences should not be taught.

Hopeful expectation: formal mathematics mechanised in “systems that explain themselves” provides experience with specifics of formal language.

Thank you for attention!
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