The image depicts how our robot Irma3D sees itself in a mirror. The laser looking into itself creates distortions as well as changes in intensity that give the robot a single eye, complete with iris and pupil. Thus, the image is called "Self Portrait with Duckling".
3D Point Cloud as ...

... vector of \((x,y,z)\)-values

- In 3DTK we have ...
  - While reading a 3D Point Cloud
    virtual void readScan(const char* dir_path,
                          const char* identifier,
                          PointFilter& filter,
                          std::vector<double>* xyz,
                          std::vector<unsigned char>* rgb,
                          std::vector<float>* reflectance,
                          std::vector<float>* amplitude,
                          std::vector<int>* type,
                          std::vector<float>* deviation);

    - Called e.g., in the function BasicScan::get()

    - Finally the data is stored in a STL-map
      std::map<std::string, std::pair<unsigned char*, unsigned int>> m_data;
3D Point Cloud as ...

... as range / intensity image

- 2D array for kinect-like sensors
- Laser scanners
3D Point Cloud as ... 

... as range / intensity image

- 2D array for kinect-like sensors
- Laser scanners
3D Point Cloud as ...

... as range / intensity image

- 2D array for kinect-like sensors
- Laser scanners
3D Point Clouds as 2D arrays (1)

- Laser scanners
  - Equirectangular projection

\[
x = \theta \\
y = \varphi
\]
3D Point Clouds as 2D arrays (2)

- Laser scanners
  - Cylindrical projection
3D Point Clouds as 2D arrays (3)

- Laser scanners
  - Cylindrical projection

\[
x = \theta \\
y = \tan \varphi
\]
3D Point Clouds as 2D arrays (4)

- Laser scanners
  - Mercator projection
    - Cannot be “constructed”, only computational principle
    - The Mercator projection is an isogonic projection, i.e., angles are preserved

\[
x = \theta \\
y = \ln \left( \tan \varphi + \frac{1}{\cos \varphi} \right)
\]
3D Point Clouds as 2D arrays (5)

- Laser scanners
  - Rectilinear
  - also “gnomonic" or “tangentplane" projection.

The primary advantage of the rectilinear projection is that it maps straight lines in 3D space to straight lines in the 2D image.

\[
x = \frac{\cos \varphi \sin (\theta - \theta_0)}{\sin \varphi_1 \sin \varphi + \cos \varphi_1 \cos \varphi \cos (\theta - \theta_0)}
\]
\[
y = \frac{\cos \varphi_1 \sin \varphi - \sin \varphi_1 \cos \varphi \cos (\theta - \theta_0)}{\sin \varphi_1 \sin \varphi + \cos \varphi_1 \cos \varphi \cos (\theta - \theta_0)}.
\]
3D Point Clouds as 2D arrays (6)

- Laser scanners
  - Rectilinear
  - also "gnomonic" or "tangentplane" projection.
3D Point Clouds as 2D arrays (7)

- Laser scanners
  - Pannini, also called Panini or "Recti-Perspective" or "Vedutismo"
    - This projection can be imagined as the rectilinear projection of a 3D cylindrical image.

- This image is itself a projection of the sphere onto a tangent cylinder.
- The center of the rectilinear projection can be different and is on the view axis at a distance of \( d \) from the cylinder axis.
- The recommended field of view for the Pannini projection is less than 150° in both vertical and horizontal directions.
3D Point Clouds as 2D arrays (8)

- Laser scanners
  - Pannini projection

\[
x = \frac{(d + 1) \sin (\theta - \theta_0)}{d + \sin \varphi_1 \tan \varphi + \cos \varphi_1 \cos (\theta - \theta_0)}
\]

\[
y = \frac{(d + 1) \tan \varphi \left( \cos \varphi_1 - \sin \varphi_1 \left( \frac{1}{\tan \varphi} \right) \cos (\theta - \theta_0) \right)}{d + \sin \varphi_1 \tan \varphi + \cos \varphi_1 \cos (\theta - \theta_0)}
\]
3D Point Clouds as 2D arrays (9)

- Laser scanners
  - Stereographic projection
    - It can be imagined by placing a paper tangent to a sphere and by illuminating it from the opposite pole.

\[
x = \frac{2R \cos \varphi \sin (\theta - \theta_0)}{1 + \sin \varphi_1 \sin \varphi + \cos \varphi_1 \cos \varphi \cos (\theta - \theta_0)}
\]

\[
y = \frac{2R (\cos \varphi_1 \sin \varphi - \sin \varphi_1 \cos \varphi \cos (\theta - \theta_0))}{1 + \sin \varphi_1 \sin \varphi + \cos \varphi_1 \cos \varphi \cos (\theta - \theta_0)}
\]

- \( R = 1 \) generates exactly the same equations as the Pannini projection and high values for \( R \) introduce more distortion.
3D Point Clouds as 2D arrays (10)

- Laser scanners
  - Stereographic projection
More Information per Pixel

- For representing a 3D point cloud as array it is advantageous to store more information per (x,y)-pixel in a panorama image (cf. `panorama.h` and `panorama.cc`)

```cpp
tuple[iReflectance] = (*it)[3]*255; //reflectance
tuple[iRange] = range; //range

if(mapMethod == FARTHEST){
    //adding the point with max distance
    if( tuple[iRange] < range ){
        tuple[iMap] = (*it);  //x, y, z
    }
}
else if(mapMethod == EXTENDED){
    //adding all the points
    cv::Vec3f point;
    point[0] = (*it)[0]; //x
    point[1] = (*it)[1]; //y
    point[2] = (*it)[2]; //z
    extendedIMap[y][x].push_back(point);
}
```
Other Representation of 3D Point Clouds

- Please consider the following
Other Representation of 3D Point Clouds

- Please consider the following
Other Representation of 3D Point Clouds

• Please consider the following
Other Representation of 3D Point Clouds

• Please consider the following
Other Representation of 3D Point Clouds

- Please consider the following
Other Representation of 3D Point Clouds

- Please consider the following
Other Representation of 3D Point Clouds

- Please consider the following

- Oc-trees represent a way to store 3D point cloud data
Further Readings

- Please consider the paper - “A Study of Projections for Key Point Based Registration of Panoramic Terrestrial 3D Laser Scans”

- Please read the paper - “Octrees for storing 3D point clouds” of the paper “One Billion Points in the Cloud – An Octree for Efficient Processing of 3D Laser Scans”

- Things to try
  - Viewing a high resolution outdoor 3D scan with reflectance values
    
    ```sh
    bin/show --s 0 --e 0 --f riegl_txt --reflectance bremen_city --saveOct
    bin/show --s 0 --e 0 --f riegl_txt --reflectance bremen_city --loadOct
    ```
Oc-trees (1)

- Every node has 8 children
Oc-trees (2)

- Empty nodes / voxels can be pruned
- Every node has 8 children

```
struct OcTree {
    float center[3];
    float size[3];
    OcTree *child[8];
    int nr_points;
    float **points;
};
```

- Definition of an oc-tree with redundant information and eight pointers to child nodes. The size of this node is 100 Bytes.
Oc-trees (3)

- Statistics of the Bremen City data set

<table>
<thead>
<tr>
<th>Leaf size (cm)</th>
<th>Mem. Size</th>
<th>Constr. time (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8560</td>
<td>1.9 kB</td>
<td>557.1</td>
</tr>
<tr>
<td>4280</td>
<td>4.8 kB</td>
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<td>1070</td>
<td>45.3 kB</td>
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<td>535</td>
<td>130.1 kB</td>
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<td>267</td>
<td>405.2 kB</td>
<td>1529.4</td>
</tr>
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<td>133</td>
<td>1.25 MB</td>
<td>1656.5</td>
</tr>
<tr>
<td>66.8</td>
<td>3.85 MB</td>
<td>1895.5</td>
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</tr>
<tr>
<td>0.52</td>
<td>2.643 GB</td>
<td>3199.9</td>
</tr>
</tbody>
</table>

- Exponential growth
Two proposed encodings of an octree node optimized for memory efficiency.

The child pointer as the relative pointer is the largest part of an octree node, but varies in size to accommodate different systems. In our implementation for 64 bit systems, it is 48 bit. Valid and leaf are 8 bit large.

Left: The proposed encoding with separate bit fields for valid and leaf. An entire node is thus contained in only 8 bytes of memory.

Right: Alternative solution resulting in a constant depth octree.
Efficient Oc-Trees (2)

• An example of a simple oc-tree as it is stored in 3DTK.
• The node in the upper left has three valid children, one of which is a leaf. Therefore, the child pointer only points to 3 nodes stored consecutively in memory. The leaf node in this example is a simple pointer to an array which stores both the number of points and the points with all their attributes.
## Efficient Oc-Trees (3)

<table>
<thead>
<tr>
<th>Leaf size (cm)</th>
<th># Nodes</th>
<th># Leaves</th>
<th>Mem. Size</th>
<th>Constr. time (ms)</th>
<th>Mem. Size</th>
<th>Constr. time (ms)</th>
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</thead>
<tbody>
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<td>1.9 kB</td>
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<td>3199.9</td>
</tr>
</tbody>
</table>
Efficient Oc-Trees (4)

- Comparison with other oc-trees

<table>
<thead>
<tr>
<th>Library</th>
<th>Node size</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>xgrt</td>
<td>$144 + x$</td>
<td></td>
</tr>
<tr>
<td>octomap</td>
<td>$72 + x$</td>
<td></td>
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<tr>
<td>PCL</td>
<td>$64 + x$</td>
<td></td>
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<tr>
<td>PCL low memory base</td>
<td>$25 + x$</td>
<td>(since ver 1.1.1. Sept. 2011)</td>
</tr>
<tr>
<td>CloudCompare</td>
<td>16</td>
<td>size of leaf node</td>
</tr>
<tr>
<td>3DTK</td>
<td>8</td>
<td></td>
</tr>
</tbody>
</table>
Using an Oc-tree for 3D Point Cloud Reduction

- Generate an oc-tree until you reached the desired voxel size
- Select the center point of each voxel for the reduced point cloud.

Or

- Select n point randomly from each voxel.

- This is implemented in 3DTK (program scan_red)

```
bin/scan_red -s 0 -e 0 -f RIEGL_TXT
    --reduction OCTREE --voxel 10 --octree 0
    --reflectance ~/dat/bremen_city/
bin/show -s 0 -e 0 -f uosr ~/dat/bremen_city/reduced
```
3D Point Cloud as ...

... vector of $(x,y,z)$-values
... as range/intensity images
... as oc-trees

- Point Cloud reduction using Oc-trees
- Now: 3D Point Cloud reduction using range/intensity images
Nearest Neighbor Interpolation
Bilinear Interpolation

- is an extension of linear interpolation for interpolating functions of two variables (e.g., x and y) on a regular 2D grid.

- $f$ is known at $(0, 0), (0, 1), (1, 0),$ and $(1, 1)$

$$f(x, y) \approx [1 - x \cdot x] \begin{bmatrix} f(0, 0) & f(0, 1) \\ f(1, 0) & f(1, 1) \end{bmatrix} \begin{bmatrix} 1 - y \\ y \end{bmatrix}.$$
Applications to 3D Point Clouds

- To Reduce an image we could
  1. Create a range image
  2. Downsample the range image (and the intensity image)
  3. Convert the range image back to a 3D Point Cloud
     - This implies implementing the inverse transformations of the image generation

```
bin/scan_red -s 0 -e 0 -f RIEGL_TXT
  --reduction RANGE --projection EQUIRECTANGULAR
  --scale 0.5 --width 3600
  --height 1000 ~/dat/bremen_city/
```
Either scale the range image and do the inverse mapping, or put the 3D points into a pixel and use interpolation

```
bin/scan_red -s 0 -e 0 -f RIEGL_TXT
  --reduction INTERPOLATE --projection EQUIRECTANGULAR
  --scale 0.5 --width 3600
  --height 1000 ~/dat/bremen_city/
```